Carnap's Tolerance and Friedman's Revenge

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September 19, 2014

Abstract

In this paper, I defend Rudolf Carnap's Principle of Tolerance from an accusation, due to Michael Friedman, that it is self-defeating by prejudicing any debate towards the logically stronger theory. In particular, Friedman attempts to show that Carnap's reconstruction of the debate between classicists and intuitionists over the foundations of mathematics in his book *The Logical Syntax of Language*, is biased towards the classical standpoint since the metalanguage he constructs to adjudicate between the rival positions is fully classical. I argue that this criticism is mistaken on two counts: (1) it fails to fully appreciate the freedom with regard to the construction of linguistic frameworks that Carnap intended his Principle to embody, and (2) Friedman's objection underestimates the extent to which the evaluation of a framework is task-relative. I conclude that Tolerance is not self-undermining in the way that Friedman claims it is. While this is a restricted conclusion – and is not a vindication of Carnap's views on logic and mathematics *tout court* – it nonetheless suggests that his tolerant perspective has been dismissed too quickly, even by his supporters.

Keywords: Rudolf Carnap, Michael Friedman, Principle of Tolerance, Logical Pluralism, Object-language/Metalanguage Distinction

Perhaps the most well-known feature of Rudolf Carnap's book *The Logical Syntax of Language* (*LSL*) is his 'Principle of Tolerance'.[1] It codifies the thought that any theory can be considered as an account of any phenomenon, as long as it is precisely specified. This is a strikingly permissive view, and the question that concerns us here is whether its permissiveness is also its undoing. Arguments to this effect have dogged the Principle since Carnap first proposed it,

*For feedback on earlier versions, I would like to thank Peter Milne, Fenner Tanswell, and the Arché WIP group, as well as audiences at LOGICA 2014, BAHPS 2014, and ECAP8.

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[1] In what follows, I will refer to the Principle of Tolerance by its full name or as either 'the Principle', or 'Tolerance'.

but in this paper I will focus on one recent example.[2] In his [Friedman (2001)], Michael Friedman argues that Tolerance cannot do the philosophical work Carnap intends it to because it biases any choice between rival positions in favor of the logically stronger view. In particular, Friedman attempts to show that Carnap's reconstruction of the debate over the foundations of mathematics in *LSL* is biased towards the classical standpoint since the metalanguage he constructs to adjudicate between the rival positions is fully classical; the Carnapian intuitionist would therefore be forced to affirm in the metalanguage what they deny at the object level. To address this problem, we must first be clear about how Carnap meant his Principle to be understood. I begin by detailing Friedman's concerns with the usual – and more narrow – understanding of Tolerance in section 1. In section 2, I outline a new interpretation of Tolerance which I call the 'wide' reading. On this understanding, there are absolutely no constraints whatsoever on what can be proposed as a linguistic framework. Because of the permissiveness of this reading, the majority of the section is occupied by an argument for treating the wide reading as the intended one. I conclude the paper by putting the wide interpretation to work, so to speak, and use it in section 3 to show how Friedman's concerns miss their mark.

1 Metalevel Tolerance and Friedman's Revenge

In his 2001 paper "Tolerance and Analyticty in Carnap's Philosophy of Mathematics", Michael Friedman raises a problem for the way that he, and many others, read Carnap's Principle of Tolerance. On his interpretation of the Principle, there are no constraints on the proposing of a linguistic framework; we are just as free to accept a language in which our background logic is constructivist as we are to accept one where it is classical. But when we consider how we might *choose between* various proposed languages, according to Friedman, problems emerge. Carnap tells us that, where before we might give philosophical arguments that one language correctly captures the nature of a concept (as, for example, logical consequence), when we attempt to work in the spirit of the Principle, we see that these debates are vexed – Carnap says these debates are "wearisome" and concern "pseudo-problems" ([Carnap (1937)], p *xiv* – *xv*). What we ought do instead is to determine the consequences of adopting the proposed languages; once we know what the consequences of each proposal are, we are in a position to make a choice. Carnap tells us that this choice will be made on the basis of the *purely pragmatic* features of the languages in question. That is, we should evaluate the languages on the basis of their relative simplicity, economy, expressiveness etc. However, things are not so easy when we get down to the details of what 'determining the consequences of adopting a proposed language' amounts to, and it is precisely here that Friedman thinks that Tolerance falls down. In this section, I take up Friedman's worry, and attempt

[2]Other philosophers who have given arguments to similar effect include: Michael Potter ([Potter (2002)]), Hilary Putnam ([Putnam (1983)], and [Ricketts (1994)]), and Eino Kalia as reported in [Carus (2007)].

to make clear its structure.

Suppose that one wants to follow Carnap's advice in determining which of these two logico-mathematical systems are best for formalizing the language of science: intuitionist or classical. Presumably, in order to establish an answer to this question, we will have to construct a suitable syntax-language which can express all the necessary concepts for *both* languages; that is, we will need a metalanguage which is capable of describing both languages so that we can adequately investigate the consequences of adopting them in this metalanguage. Moreover, this metalanguage ought be neutral with respect the differences between the two languages, otherwise we prejudge the issue. However, as Friedman notes,

> In giving a metatheoretical description of [the language of classical mathematics], we therefore need a metalanguage even stronger than the language of classical mathematics itself (containing, in effect, classical mathematics plus a truth-definition for classical mathematics). And we need this strong metalanguage, not to prove the consistency of the classical linguistic framework in question, but to simply describe and define this framework in the first place so that questions about the consequences of adopting it (including the question of consistency) can then be systematically investigated ([Friedman (2001)], pp 242 – 243).

That is, any metalanguage that is going to be adequate for describing the language of classical mathematics will have to include not only the resources to define concepts like "analytic-in-$\mathcal{L}$" and "consequence-in-$\mathcal{L}$" (where $\mathcal{L}$ is the language in of classical mathematics), but also the totality of classical mathematics itself. Moreover, since it is to contain all of classical mathematics, it will also contain the resources used by classical mathematics. So, the metalanguage we construct to judge between the intuitionist proposal and the classical one – which, recall, is supposed to be neutral between the proposed languages – is committed to resources that the intuitionist would reject, namely those used in classical mathematics. In Friedman's words:

> In order to apply the principle of tolerance, we must view [the] choice [between intuitionistic and classical languages] as a purely pragmatic decision about "linguistic forms" having no ontological implications about "facts" or "objects" in the world. [...] Accordingly, we must view the logico-mathematical rules in question, in both linguistic frameworks, as sets of purely analytic sentences. Given Carnap's own explication of the distinctions between logical and descriptive terms, analytic and synthetic sentences, however, we must have already adopted the classical logico-mathematical rules in the metalanguage. Thus, to understand the choice between classical and intuitionistic logico-mathematical rules in accordance with the principle of tolerance, we must have already built the former logico-mathematical rules into our background syntactic metaframework.

> We must have already biased the choice against the intuitionist in the very way we set up the problem ([Friedman (2001)], pp 242 – 243).[3]

In other words, in order to treat the logico-mathematical rules of the two candidate object-languages as analytic (in their respective languages), we must be able to draw the distinction between analytic and synthetic sentences in those languages. This distinction is drawn in the metalanguae, and so the relevant metalanguage must therefore have certain resources. In this particular case, as was noted above, it must have the resources of classical mathematics (otherwise it will be unable to draw the required distinction for the classical object-langauge). But, if it does, then it would seem that, again, the Carnapian intuitionist would be forced to affirm in the metalanguage the very rules she took to be in dispute at the object level. Because of the inability to neutrally frame the decision between competing frameworks, much less provide a place to stand from which we might neutrally assess the choice between them, Friedman concludes that the Principle of Tolerance is self-undermining. That is, any attempt to actually *use* the Principle, understood in the way that Friedman does, must fail to to be neutral in the way it must be to do the work that Carnap envisions for it.

2 Tolerances: Wide and Narrow

As we saw just above, if we understand the Principle in the way that Friedman does, the Carnapian project looks doomed. In this section, I distinguish two readings of the Principle of Tolerance. The first, which I argue has been the more common of the two, I call the 'narrow' reading. According to it, when Carnap says that there are "no morals in logic", he does not mean that all bets are off, so to speak ([Carnap (1937)], p 52). Rather, he means that anyone is free to propose any formal system for our *first-order* theorizing.[4] The various proposed languages are then compared by examining the consequences to which each leads. A determination about which to adopt is reached on the basis of the pragmatic merits of the proposed languages. In this section, I will show that despite the popularity of this narrow reading it is nonetheless mistaken. My strategy will be to focus on the textual evidence from *LSL*, and I will conclude from this evidence that Carnap is committed instead to an alternative understanding of the Principle whereby the volunterism and pragmatic orientation

[3] For the purposes of this paper, I set aside questions which Friedman alludes to about Tolerance and ontology. Interested readers are commended to the discussions found in [Hirsch (2009)] and [Eklund (2009)].

[4] In *LSL*, Carnap is primarily concerned with languages for mathematics, but there is nothing special about this example; in his introduction, Carnap says "The standpoint we have suggested [...] relates not only to mathematics, but to all questions of logic" ([Carnap (1937)], p *xv*), and moreover, in the first section of the book proper, he says, "The method of syntax which will be developed in the following pages will not only prove useful in the logical analysis of scientific theories – it will also help in the *logical analysis of the word-languages*" ([Carnap (1937)], p 8, original emphasis).

that Tolerance expresses extends to the full extent of the linguistic hierarchy. I call this alternative understanding the 'wide' reading, and will detail it below.

Carnap is often bold in his writing and nowhere is this tendency more apparent than in his discussions of the Principle of Tolerance. While its full formulation, given in section §17 of *LSL*, is dramatic, it is eclipsed entirely by what he says in the introduction:

> For language, in its mathematical form, can be constructed according to the preferences of any [point of view]; so that no question of justification arises at all, but only the question of the syntactical consequences to which one or other of the choices leads, *including the question of non-contradiction* ([Carnap (1937)], p *xv*, my emphasis).

Here, Carnap denies that there are any constraints on language construction outright; he even gives up consistency as a necessary condition on a language in order for it to be considered. Moreover, he also denies that one must justify one's choices. The only factor that relevant to the question of which language to adopt, Carnap says, are the consequences which accepting a candidate language leads:

> The fact that no attempts have been made to venture still further from the classical forms is perhaps due to the widely held opinion that any such deviations must be justified – that is, that the new language-form must be proved to be 'correct' and to constitute a faithful rendering of 'the true logic'.
>
> To eliminate this standpoint, together with the pseudo-problems and wearisome controversies which arise as a result of it, is one of the chief tasks of this book. In it, the view will be maintained that we have in every respect complete liberty with regard to the forms of language; that both the forms of the construction for sentences and the rules of transformation (the latter are usually designated as "postulates" and "rules of inference") may be chosen quite arbitrarily ([Carnap (1937)], pp *xiv* – *xv*).

So, according to Carnap, there is no need for a proof of a proposed language's adequacy. That is, we can suggest any language we wish for a task – "we have *complete* liberty in *every* respect" – and neither our proposed languages nor our eventual choice need be provably 'correct', whatever the salient standard of correctness might be. Finally, we look at the Principle itself:

> *In logic there are no morals.* Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments ([Carnap (1937)], p 52. Original emphasis).

Though it comes a fair way into the book, at least in comparison to the quotations from the introduction, the phrasing here is nonetheless of a piece with the tone of those early passages. What is key from our present perspective is, again, the emphasis on liberty and freedom from restrictions on which languages can be proposed. Even the last clause – "all that is required of him is that, *if he wishes to discuss it*, he must state his methods clearly, and give syntactical rules instead of philosophical arguments" – is phrased in terms of a task. It is only in the case that one wants to discuss one's form of logic that this condition applies; even the need to give syntactical rules is not an absolute. However, since we must be cautious against reading too much into the rhetoric of the introduction and the early parts of the book, and, moreover, since everything we have said so far is consistent with both the wide and narrow understandings of the Principle, we now turn to the details of Carnap's behavior in *LSL*. In order to get clear on the intended interpretaion of Tolerance we will examine two episodes from the book: firstly his treatment of intuitionism, and secondly his consistency proof for Language II.[5] These examples will serve to highlight the use that he makes of the Principle, and in each case we will examine a slightly different aspect of Carnap putting Tolerance to work.

2.1 Putting Tolerance to Work Part 1: Intuitionism

Though Carnap's discussion of intuitionism is short – it comes in section §16 of the book, and runs for only a few pages – it is highly significant. He begins by expressing frustration that no one has given a formal treatment of the intuitionistic view, and moreover that some of the intuitionists see the task of giving a formalism as unnecessary.[6] Because of this lack of a formal treatment, he claims, it is impossible to make sense of the claims over the nature of logic that intuitionists make:

> Once the fact is realized that all pros and cons of the Intuitionist discussions are concerned with the forms of a calculus, questions will no longer be put in the form: "What *is* this or that like?" but instead we shall ask: "How *do we wish to arrange* this or that in the language to be constructed?" or, from the theoretical standpoint: "What consequences will ensue if we construct a language in this or that way?" ([Carnap (1937)], pp 46 – 47, original emphasis).

Of course, this "realization" just is the Principle of Tolerance applied to case of the debate between intuitionists and classical logicians. By the lights of the Principle, since the intuitionists have not yet produced a formal treatment of their view, or at least not one that Carnap deems adequate for discussion,

[5]In sections 2.1 and 2.2 below, I assume familiarity with both Language I and Language II from *LSL*. Carnap gives the basic setup of I in Part I of *LSL* and the setup for II in Part III; a useful commentary on these frameworks can be found in the introduction to [Wagner and Beaney (2009)].

[6]Carnap does note Heyting's book as an interesting first attempt, though does not have more to say about it than that at this stage of the book ([Carnap (1937)], p 46). When he takes the issue up again, however, he complains that Heyting's formalization is inadequate because the distinction between object-language and syntax-language is not drawn ([Carnap (1937)], pp 249 – 250).

then for the purposes of determining which language should form the basis of our mathematical reasoning, or which logical principles can be accepted in our mathematical practice, we are free to make precise their claims in any way we think fit. Carnap puts the point this way:

> It is in order to exclude [indirect proofs which lead] to an unlimited, non-constructive existential sentence that Brouwer renounces the so-called *Law of the Excluded Middle*. The language-form of I, however, shows that the same result can be achieved by other methods – namely, by means of the exclusion of the unlimited operators. [...] Thus Language I fulfills the fundamental conditions of Intuitionism in a simpler way than the form of language suggested by Brouwer (and partially carried out by Heyting) ([Carnap (1937)], p 48, original emphasis).

Since there are many ways we might give a precisely specified linguistic framework which meets the "fundamental conditions" of some philosophical view – in this case intuitionism – there must be a way to decide between these proposals. The way to adjudicate between them indicated by Carnap in the quotation, just as the Principle says it should be, is by comparing their pragmatic features. In this case, he thinks that Language I achieves the same aims as Heyting's proposed formalization of Brouwer's philosophical view, but that I does so in a simpler way. On that basis, and only on that basis, it is to be preferred.

Returning to the question of the proper interpretation of the Principle of Tolerance, what stands out about Carnap's treatment of intuitionism in *LSL* is its early placement in the book. Though it comes before the formal statement of the Principle, it serves to set the stage for that statement.[7] His discussion is, in that way, a case study in how to *act* in accordance with Tolerance. That is, instead of engaging with debates over the nature of negation, or of whether quantification is restricted, instead he gives the rules for a framework that he claims achieves the same aims. Anyone who disagrees is invited to do the same, and the choice between Carnap's Language I and any alternative framework will be made on the basis of the consequences of their adoption. Another part of *LSL* that helps make clear the way in which Carnap understood the Principle is his purported consistency proof, which we examine in section 2.2 below. However, before moving to the question of consistency, we pause briefly to discuss what it is, exactly, that Carnap is tolerant of.

2.1.1 The Limits of Carnapian Tolerance

Carnap's reconstruction of the intuitionist position is a drastic departure from anything that Brouwer would have accepted. To begin with, Carnap's focus on formalizations of a language is antithetical to Brouwer's perspective.[8] Carnap is

[7] In the fist edition, the statement of the Principle comes in the very next section. For the second edition, Carnap inserted a short part, section §16*a*, on Wittgenstein's theory of identity.

[8] See [Mancosu (1998)], p 2.

perfectly aware of this situation, and, moreover, shows no hesitation in ignoring Brouwer's view:

> We hold that the problems dealt with by Intuitionism can be exactly formulated only by means of the construction of a calculus, and that all the non-formal discussions are to be regarded merely as more or less vague preliminaries to such a construction ([Carnap (1937)], p 46).

This dismissal of all non-formal discussion before the construction of a linguistic framework is at the heart of the Principle. It is what Carnap means when he says that one must "[...] give syntactical rules instead of philosophical arguments" ([Carnap (1937)], p 52). In other words, what the Principle enjoins us to be tolerant of is precisely formulated languages which are proposed for adoption, and *not* any philosophical justifications for those proposals. In the absence of a proposed formal language, these philosophical considerations will be completely superfluous because it will not be clear which position they support; that is, since there are many different logics that could be thought to be the formal precisification of Brouwers' claims, and so those philosophical claims do not settle the issue of which of the formal presentations we should pick. Carnap shows this implicitly by producing a linguistic framework, namely Language I, which he claims captures the spirit of intuitionism, despite its obvious departures from the philosophical claims that Brouwer makes.[9] With this in mind, we now turn to the second case study in Carnap's use of the Principle.

2.2 Putting Tolerance to Work Part 2: Consistency

Carnap addresses the question of consistency several times in *LSL*. The first two times are in the introduction, where he dismisses the worry that allowing a language to represent its own syntax will result in contradictions along with the demand that a proposed language be consistent in order to be considered for adoption. Despite this early dismissal, Carnap nonetheless offers a proof of the consistency of Language II in section §34*i*, and remarks on the issue again in section §59, "General Syntax". I will largely constrain my discussion to the earlier section, §34*i*, because the two sections are very similar.

The proof that Carnap gives for Language II is somewhat laborious, and we will not be concerned to cover the details. More interesting for our current investigation are Carnap's comments after the proof on the relationship between his result and Hilbert's program. After a few remarks to the effect that his term 'definite syntactical concepts' is approximately equivalent to Hilbert's 'proof with finite means', he says,

> Whether with such a restriction [to the use of only definite syntactical concepts in a consistency proof for classical mathematics], or

[9]For example, the Law of the Excluded middle is valid in I, while rejected by Brouwer. See [Carnap (1937)], p 48 and p 34 Theorem 13.2.

> anything like it, Hilbert's aim can be achieved at all, must be regarded as at best very doubtful in view of Gödel's researches on the subject (see §36). [...] The proof we have just given of the non-contradictoriness of Language II, in which classical mathematics is included, by no means represents a solution to Hilbert's problem. Our proof is essentially dependent upon the use of such syntactical terms as 'analytic', which are indefinite to a high degree, and which, in addition, go beyond the resources at the disposal of Language II ([Carnap (1937)], p 129).

This passage is critical to the wide understanding of Tolerance. One of the tasks that Carnap set himself in *LSL* was to show that all of classical mathematics can be formalized, and that the language constructed to do it does not contain any contradictions. The latter challenge is completed by the proof, at least in a somewhat attenuated sense. Strictly speaking, what Carnap shows, as he comments somewhat later on, is that "*everything mathematical can be formalized, but mathematics cannot be exhausted by one system*; it requires an infinite series of ever richer languages" ([Carnap (1937)], p 222, original emphasis). So, the first challenge is impossible – there is no single language which can serve for the formalization of *all* of mathematics. We might think that because of this fact, the consistency proof for II only shows the consistency of that part of mathematics which can be formalized in II. But, in order to complete the proof, Carnap has had to make use of resources that go beyond those available in II, namely the concepts 'analytic-in-II' and 'consequence-in-II' which are both indefinite in II. In the proof, these concepts are used in the metalanguage where they may very well be definite.[10] This not only shows, as was remarked above, that the consistency of mathematics cannot be demonstrated because there is no single language which can capture all of mathematics, it also shows that any purported proof of a language's consistency is only ever a proof of *relative* consistency. That is, we can only ever show that, by the lights of one language, some other language is consistent; in the case of Carnap's proof, we show that II is consistent if the metalanguage for II is.

What attracts our attention now, however, is his frank admission that the proof offers no absolute certainty, as well as his total lack of apparent concern over the effect that this has on the status of the proof. Carnap says:

> Our proof is essentially dependent upon the use of [...] syntactical terms [...] which are indefinite to a high degree, and which, in addition, go beyond beyond the resources at the disposal of Language II. Hence, the significance of the presented proof of non-contradictoriness must not be over-estimated. Even if [our proof] contains no formal errors, it gives us no absolute certainty that contradictions in the object-language II cannot arise. For, since the

[10] Of course, the metalanguage for II (which we may as well call III) will have corresponding concepts which are indefinite in *it*. We know, for example, that the concept 'analytic-in-III' will be indefinite in III for analogous reasons. This fact is related (indeed is nearly equivalent) to Tarski's Theorem on the indefinability of truth. See [Procházka (2006)] for a discussion.

> proof is carried out in a syntax-language which has richer resources than Language II, we are in no wise guaranteed against the appearance of contradictions in this syntax-language, and thus in our proof ([Carnap (1937)], p 129).

This is all said without further comment. By his silence, Carnap seems to suggest that, were we to have concerns about whether or not the proof is good, then we are free to construct a third language, capable of serving as a metalanguage for the metalanguage for II, and then to carry out a similar proof of the consistency of II's metalanguage in it. This new proof will have the same epistemic character as our original consistency proof, namely that it does not guarantee us against paradox in the language in which we conduct the proof. What must lay behind Carnap's rather *laissez faire* attitude towards consistency proofs is, I claim, wide Tolerance. That is, since we are just as free to construct metalanguages as object-languages, we can always find a place to stand from which we can investigate any question that might interest us.

The Principle of Tolerance, on either the wide or the narrow interpretation, already licenses Carnap's dismissal of foundational concerns. That is, the complaint that our proof does not guarantee absolute consistency, but only consistency relative to some other theory, has no bite for him because leaving aside such demands for absolutes is just what the Principle enjoins us to do. However, what we see in the quotation is not simply Carnap's anti-foundationalism, but the thorough-going nature of his understanding of Tolerance. Recall that, for Carnap, statements about which language to choose can only be evaluated relative to a stated goal. This too is consistent with either the wide or narrow interpretation of the Principle. So, according to either interpretation, if our goal is to show that our object-language is consistent, then we need to construct a syntax-language with the appropriate resources (as, for example, being able to express the concept "analytic-in-I"). This point generalizes, and choice of task will have consequences for the way in which we should construct the syntax-language. Carnap puts the point directly himself in section §45:

> Our attitude towards the question of indefinite terms conforms to the principle of tolerance; in constructing a language we can either exclude such terms (as we have done in Language I) or admit them (as in Language II). It is a matter to be decided by convention. If we admit indefinite terms, then strict attention must be paid to the distinction between them and the definite terms; especially when it is a question of resolubility. Now this holds equally for the terms of syntax. [...] Some important terms of the syntax of transformations are, however, indefinite (in general) [...] ([Carnap (1937)], pp 165).[11]

So far, everything that Carnap has says is consistent with both the wide and the narrow interpretation. However, he continues on to say:

[11] Friedman also notes this passage at [Friedman (2001)], p 227.

> If we wish to introduce these [indefinite] terms also, we must use an indefinite syntax-language (such as Language II) ([Carnap (1937)], pp 165 – 166).

So, we now see that just as with the object-language, there are no morals at the metalinguistic level either.[12] That is, we are free to construct our syntax-language in any way we see fit, with the recognition that some ways of doing so may fare 'better' than others for particular tasks. This claim, namely that Carnap not only intended for Tolerance to apply at the object level, but at the level of syntax-languages as well, just is the distinction between the wide and narrow interpretations. So, while either interpretation can accommodate the task-relativity of Tolerance at the object level, it is wide Tolerance that enjoins us to follow the Principle all the way up the linguistic hierarchy.

3 Wide Tolerance and the Boundless Ocean

I begin with an obvious, but as I will show ultimately ineffective, route for Carnap to escape Friedman's arguement. Friedman suggests the Carnap might be available is to adopt a weaker, intuitionistically acceptable metalanguage (as, for example, one that excludes unlimited universal quantification), instead of the full-blooded classical one he in fact adopts in *LSL*. In his 2007 paper, Ricketts takes a similar line against Friedman's concerns. He says:

> There is nothing inconsistent or untoward in an advocate of weak logic for the language of science using a strong meta-language both to set forth her favored language and to compare it with other proposed languages for science.[13] Nevertheless, a tolerant advocate of a weaker logic *may* balk at the use of a strong meta-language. [...] The refusal of our advocate of a weaker logic to use a strong meta-language does not, however, close off the prospect of metamathematical comparisons of calculi. In such circumstances, the discussants will have to restrict themselves to those descriptions of the languages under consideration that are available in a meta-language they all share ([Ricketts (2007)], p 219. Original emphasis.).

Ultimately, and correctly in my view, Friedman finds this proposal lacking. While it would allow for the exploration of the consequences of adopting the two competing frameworks without prejudice to one over the other, what he says it will not do is allow for a "[...] sharp contrast between *merely* pragmatic questions of 'linguistic form' having no ontological import, on the one side, and genuine theoretical claims, on the other" ([Friedman (2001)], p 244, original emphasis). To see why this is the case, it is helpful to recall the reason we adopted the full strength of classical mathematics in the metalanguage in

[12]Ricketts makes a similar point in his [Ricketts (2007)], p 219.

[13]Ricketts' "languages for science" are what I have called "object-languages" in this paper; what is meant by both expressions is the languages used for our first-order theorizing.

the first place, namely to be able to make use of indefinite notions like analyticity. These notions are what serve to mark the distinction between the genuine theoretical questions, and those which are settled by linguistic conventions. So, if we adopt a metalanguage which is too weak to characterize such notions in all the languages under consideration, then while we might get the intuitionist back into the game, so to speak, we may lose the ability demarcate the boundaries of the field of play. So it would seem that the obvious maneuver – taking the strongest mutually agreeable language as metalanguage – will not be a solution in general. But there is another solution available, and it is to this that we now turn.

Friedman thinks that the tension his argument reveals in Carnap's Tolerance is ultimately fatal to the program in *LSL*.[14] However, I contend that the tension is merely apparent, and stems from a misconstrual of Tolerance as requiring the narrow reading. In particular, he has neglected to note a shift in the salient task that is critical to seeing that Tolerance is in no way undermined by the use of intuitionistically unacceptable resources in the metalanguage; however, this shift is only noticeable once one thinks that Tolerance extends to metalanguages as well.[15] Recall that for Carnap languages are evaluated relative to a particular task. To give two examples, in the case that Carnap is concerned with in *LSL* where our concern is to give a logical foundation for classical mathematics, then we ought pick a language which embraces non-constructive proof techniques. Conversely, were we to be concerned to ensure that we can decide for each numerical predicate whether it applies to a given number or not, then we ought pick a language which is constructed along the lines of Carnap's Language I.

Returning to the case at hand – that is to deciding between classical and intuitionist mathematics – what we will need to do is to construct a language that allows us to investigate the consequences of adopting each proposed language without prejudging the issue. The metalanguage so constructed need not be the same one in which we might, for example, investigate whether these same object-languages are consistent; after all, the task at hand has changed. Tolerance runs all the way up the linguistic hierarchy on the wide reading, and therefore so does task relativity. Friedman's argument assumes that there needs to be a single metalanguage, chosen once and for all; for him, it not only serves as a place to stand when considering questions about a particular language, it is also supposed to simultaneously be a perspective from which we adjudicate all disputes over the form that a language should take. But, this insistence on a single metalanguage for these disparate tasks is precisely the kind of absolutism that Carnap sought to combat with Tolerance.

There is another way in which Friedman's argument does not hit its mark. As he points out, if we want to prove facts about our object-languages (such as, for example, their consistency or completeness), we must do so in some language

[14][Friedman (2001)], p 244.

[15]The arguments I make in this section have some similarity to arguments that Ricketts makes in his [Ricketts (2007)], especially in section III of that essay. There are differences, however. In particular, my focus here is on how the two facets of Tolerance interact to defuse certain objections; Ricketts simply states that these objections have no bite.

or other. This language must have certain resources in order for the proofs to go through, as we have noted several times. We then use the results of this investigation of these languages in order to come to a determination of which of the two to adopt as the language of our first-order theorizing. Friedman claims that this prejudices the decision of which object-language to adopt because we have made essential use of resources in the metalanguage that proponents of weaker logics do not accept as valid. It is this last step that is problematic from Carnap's perspective; it relies on a notion of validity *simpliciter* which he rejects. On Carnap's picture, inferences are only valid (or invalid) relative to a particular language, and so the fact that one treats an inference as valid in one language does not entail that one must treat it as valid in every language. Though this point is obvious in the case of different object-languages, as the case at hand of classical and intuitionistic logics illustrates, it is somewhat more subtle when examining the case of an object-language and its metalanguage. However, Carnap's view is that metalanguages are constructed in just the same way as object-languages are. This means that just as with object-languages, the validity of inferences in metalanguages are language relative. So, accepting certain inferences for the task of investigating the consequences of adopting a language does not thereby commit one to the unlimited validity of those inferences. That is, the intuitionistically inclined Carnapian can still entertain the notion that inferences like double negation elimination and unrestricted universal quantification are invalid in our mathematical reasoning quite independently of accepting them for investigating the consequences of rejecting those inferences on our mathematical practice. In this way, the notion of validity in the metalanguage floats free from the notion of validity in the object-language, and conversely.

In this paper, I have developed a reading of Carnap's Tolerance which I dubbed the 'wide' reading. According to it, as I argued, there are no constraints whatsoever on the construction of languages, and therefore no constraints on what languages we can consider as potential languages to adopt for formalizing some practice. Moreover, the wide reading ensures that the task-relativity of our evaluation of languages applies at the level of metalanguages in just the same way it does at the level of object-languages. I showed that this wide reading is the most plausible way to understand Carnap's intended meaning for the Principle of Tolerance by closely examining his uses of the Principle in *LSL*. Finally, I turned attention to an argument by Michael Friedman, namely that Tolerance is self-undermining due to the prejudicial nature of the metalinguistic considerations necessary to assess proposed linguistic frameworks. I considered a possible route of escape suggested by Ricketts, namely to adopt a weaker but mutually agreeable metalanguage from which to settle disputes, but concluded that it left the Tolerance-inclined logician in a difficult place: unable to define some of the languages that one might want. However, I offered an alternative suggestion, one that does not face this problem. It relied on the wide interpretation of Tolerance, and made essential use of the task-relativity of language selection.

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